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Application of a Novel Optimal Control Algorithm to Low-Thrust Trajectory Optimization

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An application of the new optimization algorithm Static/Dynamic Control (SDC) to the design of low-thrust interplanetary trajectories is presented. The interplanetary trajectory is integrated with a multi-body force model and may include gravity assists. Engine operation is modeled as finite burns. As a result, SDC based optimization can be used for high fidelity design.

In addition to SDC's capability as a high fidelity design tool, SDC optimization provides several important features which are useful for preliminary trajectory design. A novel feature of the SDC approach is its ability to locate favorable intermediate flybys. It is not necessary to specify which intermediate flyby bodies will be used for gravity assists. This is in contrast to many existing optimization methods. Another useful feature is that SDC does not require a good initial trajectory guess to begin the optimization. SDC's dual ability to begin with poor guesses and locate favorable intermediate flybys results in the identification of non-obvious, and highly efficient trajectories.

SDC is a general optimization method that is distinct from both parameter optimization and the Calculus of Variations. The SDC optimization algorithm is based in part on the Hamilton, Bellman, Jacobi dynamic programming equation¹. Unlike traditional differential dynamic programming methods, SDC is constructed to solve highly nonlinear and non-convex problems with a dual dynamic and parametric structure. Two distinct classes of optimization variables are recognized. The first is the dynamic class which are functions of time (for example, the thrust vector as a function of time). The second variable class is the static class which can be thought of as parameters in the ordinary parameter optimization sense (for example the launch date or launch V_∞ are parameters, not functions of time.) Optimal solutions generated by SDC satisfy both the necessary and sufficient conditions of optimality.

Results produced by SDC for several test problems are compared to results produced by two other optimization programs. One program is based on the calculus of variations, the other is based on parameter optimization. The test problems include an Earth launch to Mars flyby; Earth launch to Mars flyby, to a flyby or rendezvous with the asteroid Vesta; Earth launch to Venus flyby to Mercury rendezvous; and Earth launch to Venus flyby to Mars flyby to Jupiter flyby. The test problems feature solar electric propulsion with a specific impulse that is a function of the engine throttle level. The objective is to maximize final mass taking into account a launch vehicle performance curve and propellant usage. The results from all three programs agree closely.

Some of the results for the Earth launch to Venus flyby to Mercury rendezvous problem are provided in Table 1. The optimal trajectory for this test problem as determined by SDC is plotted in Figure 1. The engine model is based on a single NSTAR 30-cm ion thruster⁶. The solar array power output at 1 AU is $P_o = 1.5$ kW and the spacecraft bus requires a constant 200 W which is not available to the thruster. With this array, the engine can operate at it's maximum rated thrust only for radii below 0.64548 AU. In the base case, the launch date is held fixed at August 29, 2002, the Venus flyby date is held fixed at February 11, 2003, and the arrival date is held fixed at December 24, 2004. Total flight time is 847 days. Other comparisons involve releasing the launch, Venus flyby, and arrival dates. The launch vehicle used is a Delta 7326 with a 10% launch vehicle contingency. The launch V_∞ magnitude and direction are free. The objective is to maximize final mass.

SDC is compared to the programs SEPTOP² and CLSEP³, both developed by the Jet Propulsion Laboratory. The program SEPTOP is a low-thrust optimization program based on the Calculus of Variations. SEPTOP is the successor program to the well known program VARITOP⁴. Both SEPTOP and VARITOP have been used extensively by the Jet Propulsion Laboratory to design a variety of low-thrust missions. The program CLSEP is a low-thrust optimization program based on nonlinear parameter optimization. CLSEP parameterizes the problem by dividing the trajectory into a series of legs. CLSEP uses the nonlinear programming software SNOPT⁵ to solve the resulting problem. Both SEPTOP and CLSEP propagate the trajectory assuming the only gravitating body

is the Sun. The flyby of Venus is modeled as an instantaneous rotation of the V_{∞} vector at Venus's center. The SDC method propagates the trajectory assuming both Venus and the Sun are gravitating. The flyby of Venus is modeled using multi-body propagation.

Table 1 indicates that all three programs produce similar optimal final mass values. The SDC solution is closer to the SEPTOP solution than to the CLSEP solution in terms of both final mass and the trade off between propellant usage and launch energy c_3 . The three programs are not expected to produce identical results due to differences in the way each method represents engine operation, flybys, and planet locations.

Program	Propellant (kg)	c_3 ($\frac{km^2}{s^2}$)	Final Mass (kg)	Venus Flyby Radius (km)
SEPTOP	179.55	7.4209	316.01	6,951
CLSEP	166.81	8.7694	312.76	6,352
SDC	177.58	7.5783	316.08	6,702

Table 1: Summary of solutions obtained for the Earth launch - Venus flyby - Mercury rendezvous problem. The Venus flyby radius is measured to Venus' center.

The starting trajectory provided to the SDC optimization program was poor. The starting trajectory consisted of a simple inward spiral (thrust directed opposite velocity). The spiral results in a spacecraft location more than 20 million kilometers from Venus on the Venus flyby date. In addition, the spiral fails to match Mercury's position at the arrival date by more than 100 million kilometers. Despite the poor initial trajectory, SDC converges readily. Convergence for SEPTOP was very difficult for this problem. A great deal of user intervention was required to get SEPTOP to converge.

In order to perform the comparisons between SDC and the two programs SEPTOP and CLSEP, the test problems require fixed intermediate flyby sequences. Both the parameter optimization and the calculus of variations methods require the intermediate flyby sequence to be given and fixed. The SDC method does not require the intermediate flyby sequence to be given and fixed. Examples of SDC's ability to identify favorable intermediate flyby sequences are presented.

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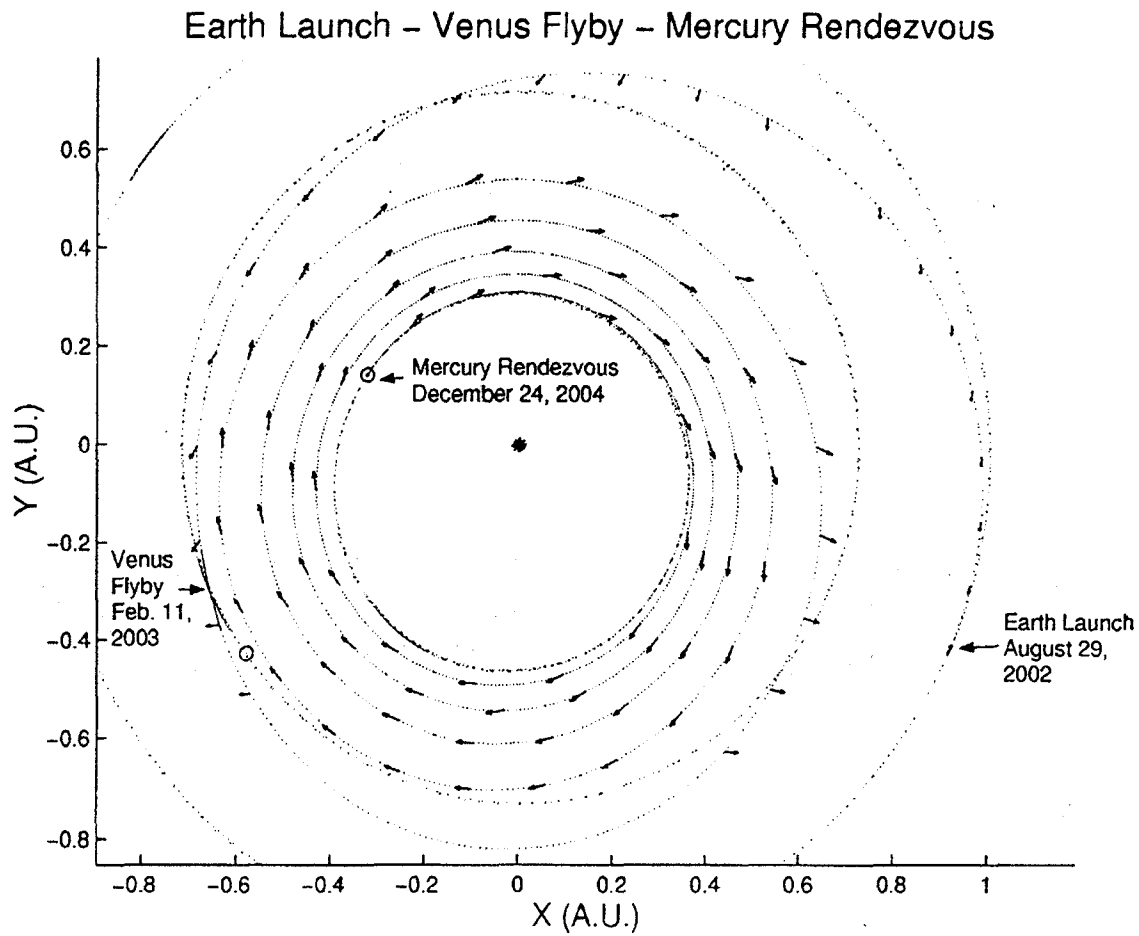


Figure 1: Optimal trajectory for the Earth launch - Venus flyby - Mercury rendezvous problem. The arrows along the spacecraft trajectory indicate the thrust direction.

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